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Technical Notes

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Similarity Relation for Maximal Gas Compression by Strong Ionizing Shocks

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Introduction

PROPAGATION of strong shock waves in inert gases resulting in the creation of a partially ionized plasma behind the shocks is of interest for physics of shock waves,[1] high-temperature gasdynamics,[2] as well as for various applications. One can point out the use of a shock-heated plasma of inert gas as a source of uv emission,[3] the employment of a moving argon plasma created behind the shock wave as a working medium for the explosive MHD generator,[4] and the shock-wave formation of supersonic plasma jets for propulsion.[5] For such applications, knowledge of the physical characteristics of the gas behind the shock, such as its density and velocity, is of great importance in their designing stage.

In this Note, the behavior of the dependence of $\eta = \rho_2/\rho_1$ as a function of M_1 is analyzed (here and below the subscripts 1 and 2 refer to the state of the gas ahead of the shock and the equilibrium region behind it, respectively). For the maximal compression degree $\eta_m = \max \eta(M_1)$ we establish a similarity law indicating that the quantity η_m represents a universal function depending on the unique combination of the initial concentation of atoms, the first ionization potential, and the ion-atom statistical sum ratio.

We consider a shock that propagates in a nonionized monatomic polytropic gas. The conservation laws that connect the physical parameters in region 1 and in region 2 yield the following relationship:

$$\eta = \frac{1 + 4P(\eta, \theta)}{4 + P(\eta, \theta) - z\alpha(\eta, \theta)}$$

$$P = \frac{p_2}{p_1}, \qquad \theta = \frac{T_2}{T_1}, \qquad z = \frac{2I}{kT_1} \tag{1}$$

where I is the ionization potential, $p_1 = n_1kT_1$, n_1 is the concentration of neutral atoms in the cold gas ahead of the shock, and k is Boltzmann's constant. The equation of state for the partially ionized plasma behind the shock is given by

$$P(\eta, \theta) = \eta\theta[1 + \alpha(\eta, \theta)] \tag{2}$$

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The ionization degree α is determined by the Saha equation

$$\alpha(\eta, \theta) = \sqrt{f^2/4 + f} - \frac{f}{2}$$

$$f = b(n_1, T_1)\theta^{3/2}\eta^{-1} \exp\left(-\frac{z}{2\theta}\right) \tag{3}$$

$$b(n_1, T_1) = \frac{g}{n_1}\left(\frac{m_e kT_1}{2\pi\hbar^2}\right)^{3/2}, \qquad g = 2\sum_i \Big/ \sum_a$$

Here, m_e is the mass of an electron, $\hbar = h/2\pi$ is Planck's constant, and Σ_i and Σ_a are the statistic sums for the ion and the atom, respectively. The latters are assumed to be constant. An implicit dependence $\eta(P)$ determined by Eqs. (1–3) represents a nonmonotonic function. The compression degree η as a function of P has a maximum $\eta = \eta_m$ at a certain value of $P = P_m$. For conditions close to the maximum compression degree we assume that the gas behind the shock is almost fully ionized, and hence

$$\theta_m >> 1, \qquad 1 - \alpha_m << 1 \tag{4}$$

where θ_m and α_m are the values of θ and α at $\eta = \eta_m$. These assumptions are confirmed by the results of the numerical calculations presented below.

Under conditions (4), η_m is asymptotically given by

$$\eta_m = 4 + \xi \tag{5}$$

where ξ is the solution of the following equation:

$$\xi^{3/2}\left(\xi + \frac{3}{2}\right)(\xi + 2)\exp \xi = Q$$

$$\xi \equiv \frac{z}{2\theta_m} = \frac{I}{kT_{2m}}, \qquad Q = \frac{g}{n_1}\left(\frac{m_e I}{2\pi\hbar^2}\right)^{3/2} \tag{6}$$

Results

Figure 1 illustrates the behavior of numerically calculated shock adiabatics $\eta = \eta(M_1)$. Three curves in this figure correspond to three different gases: xenon, argon, and neon at $T_1 = 300$ K, and at different initial pressures. The latters are picked so that the parameter H [see Eqs. (6) and (7)] is the same for all three gases.

Comparison of the theory with existing experiments for argon was made. The results are presented in Fig. 2, which shows the calculated dimensionless temperature in the equilibrium region $\theta = T_2/T_1$ as a function of M_1, together with the experimental data of Pinegre[6] as presented in Ref. 1. One can see that the proposed theoretical model is in good agreement with experiment, in spite of the fact that the theory takes into account only one mechanism of the energy losses, namely, the first ionization.

Asymptotic analysis based on Eq. (6) shows that for monatomic gases the maximal compression degree η_m and the corresponding normalized temperature $\tau_m = 1/\xi$ depend only on the parameter

$$H = gI^{3/2}/n_1 \tag{7}$$

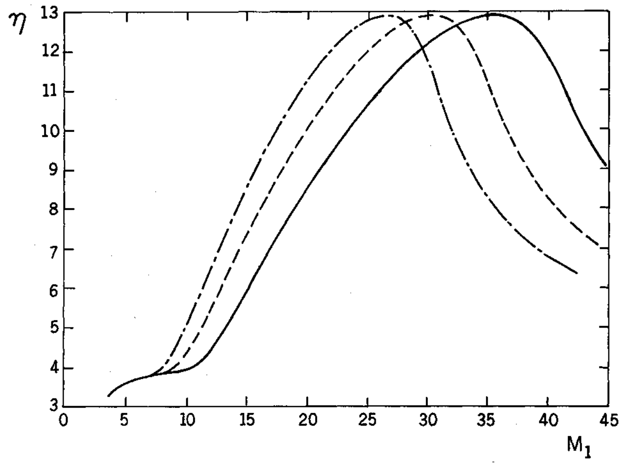

Fig. 1 Calculated shock adiabatics for different gases under the condition $H = gI^{3/2}/n_1$ = const, T_1 = 300 K. Solid line: neon, p_1 = 3.58 Torr; dashed line: argon, p_1 = 2.04 Torr; dot-dashed line: xenon, p_1 = 1 Torr.

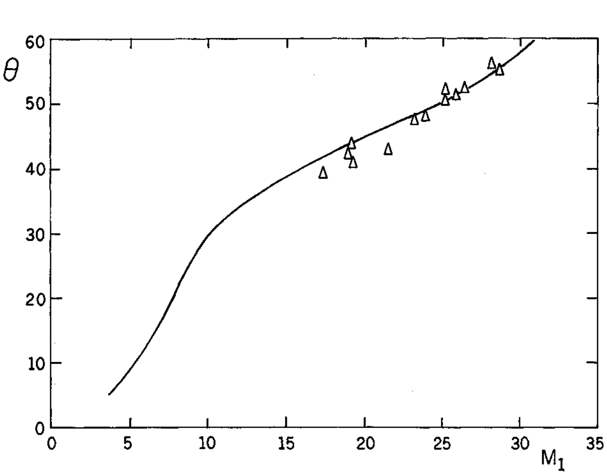

Fig. 2 Temperature ratio across the ionizing shock $\theta = T_2/T_1$ as a function of M_1 for argon at T_1 = 300 K and p_1 = 1 Torr. The solid line is the theoretical dependence $\theta(M_1)$ calculated from Eqs. (1–3). The triangles are the experimental data by Pinegre.[1,9]

characterizing the gas under consideration. The functions $\eta_m(H)$ and $\tau_m^{-1} = \xi(H)$ are universal for pure monatomic gases. This property represents a similarity law for the maximal compression degree created by ionizing shocks. The plots of $\eta_m(q)$ and $\tau_m(q)$, where $q = lgQ$, calculated according to Eqs. (5) and (6) are shown in Fig. 3. The quantity q is defined as

$$q = lgQ = 4.96 + lg(I^{3/2}g/\bar{n}_1) \tag{8}$$

In this formula the ionization potential I should be taken in electron volt (eV), and $\bar{n}_1$ is calculated as $\bar{n}_1 = 3 \times 10^{-17} n_1$, where n_1 is the concentration of atoms in cm^{-3}, so that the value $\bar{n}_1 = 1$ corresponds to the initial gas pressure $p_1 = 1$ Torr at the initial temperature T_1 = 300 K.

Lowering of the initial gas density leads to the increase in the maximal degree of compression. We further note that among all inert gases the maximal value of η_m is reached in neon having the highest value of $gI^{3/2}$, and not in helium having the highest value of ionization potential. For a given gas, the temperature T_{2m} corresponding to the maximal degree of compression is a growing function of the initial gas density ρ_1. Under the conditions [Eq. (4)] the shock Mach number M_{1m} providing for the maximal value of η is given by

$$M_{1m} \approx \frac{3z\eta_m^2}{5(\eta_m - 1)(\eta_m - 4)} \tag{9}$$

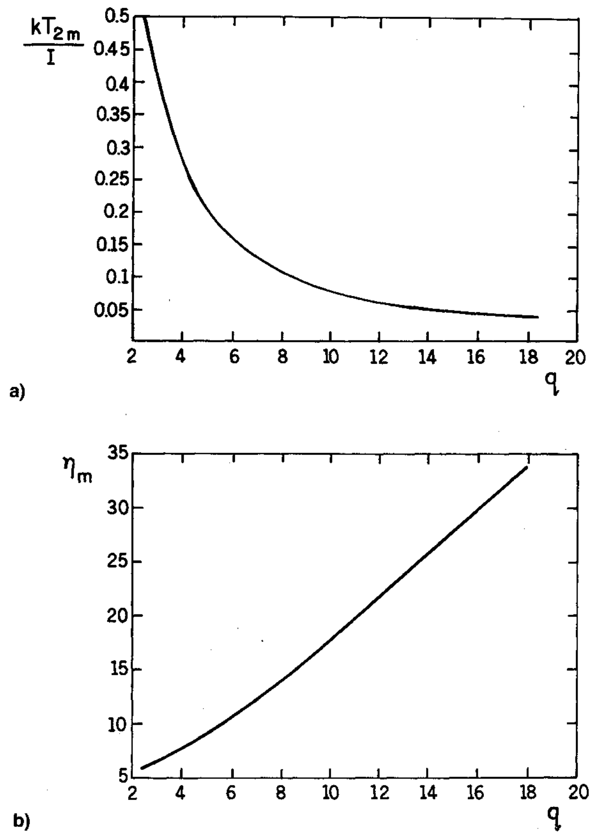

Fig. 3 a) Normalized temperature $\tau_m = kT_{2m}/I$ corresponding to the maximal degree of compression as a function of parameter q defined in Eq. (8) and b) maximal degree of compression η_m as a function of parameter q. Both curves are calculated from the asymptotic theory based on Eqs. (5) and (6).

The similarity law predicted by the asymptotic relationships (5) and (6) is in very good agreement with numerical calculations of the shock adiabatic based on the complete system of Eqs. (1–3). It is clearly seen from Fig. 1, in which the calculated maximal compression degree η_m takes one and the same value for three different gases characterized by one and the same value of the parameter H defined by Eq. (7). The quantitative comparison of the values of η_m obtained on the basis of Eqs. (1–3) with the values of $\eta_m^{(a)}$ found from the asymptotic Eqs. (5) and (6) shows that the accuracy of the asymptotic theory is fairly satisfactory. Thus, under conditions corresponding to the numerical calculations presented in Fig. 1, the quantity $\eta_m^{(a)}$ equals 13.25, whereas the exact value of η_m [as obtained separately for each gas from Eqs. (1–3)] is 12.92. For the shock adiabatics shown in Fig. 1, the values of M_{1m} are equal to 26.9, 30.4, and 36.1 for Xe, Ar, and Ne, respectively, while the asymptotic formula (9) gives for these gases at $\eta_m = \eta_m^{(a)} = 13.25$ the values of M_{1m} as follows: 29.5, 33.7, and 39.4.

In the laboratory frame of reference, in which the cold gas ahead of the ionizing shock is at rest, while the shock wave propagates with velocity W, the velocity u_2 of flow behind the shock can be determined as $u_2 = \beta W$, $\beta = \beta(M_1; T_1, p_1) = 1 - \eta^{-1}$. Since for a given initial state of the gas, the dependence $\eta(M_1)$ has a maximum at certain value of $M_1 = M_{1m}$, the dependence $\beta(M_1)$ also has a maximum at $M_1 = M_{1m}$. When ionization is neglected, $\eta < 4$, and the value of β cannot exceed 0.75. The effect of ionization is displayed in increasing the maximal value of β. Thus, for the conditions corresponding to the shock adiabatics shown in Fig. 1, the maximal value of β equals 0.92, so that the acceleration of the gas behind an ionizing shock is higher than predicted by classical gasdynamics.

Discussion

The theoretical model of an ionizing shock employed in this Note is fairly simple and can be used for calculating the shock adiabatics in monatomic gases and the maximal degree of gas compression behind the shock. Although the model takes into account only one mechanism of energy losses related to the ionization of neutral atoms, it results in good agreement with the experiment as follows from the comparison shown in Fig. 2. This means that the incorporation in the model of other energy loss mechanisms, such as the loss of energy by excitation of electron levels and by radiation cooling would be hardly expedient for calculating the ionizing shock adiabatic, at least, its increasing section. On the other hand, it should be noted that the tail part of the descending section of the shock adiabatic, for which kT_2 is of the order or larger than the difference $I_2 - I$, where I_2 is the second ionization potential, cannot be described satisfactorily by the present model. The reason is that in this region of equilibrium temperatures and corresponding shock Mach numbers, the ionization of single and multicharged ions should occur.

Although the fact that the compression degree by ionizing shocks in monatomic gases may considerably exceed the classic limiting value for gasdynamic shocks $\eta_m = 4$ was known in the literature (e.g., Refs. 1, 7, 8), the conditions providing for the maximal compression degree by such shocks and, furthermore, the similarity relation for η_m, were not clarified in earlier works. The reason for the occurrence of a maximum for the dependence $\eta(P)$ [or $\eta(M_1)$] is that the endothermic reaction of ionization leads to a drop of temperature and to a growth of density in the relaxation zone, so that the value of the compression degree passes through the classic limit $\eta = 4$ when M_1 increases. The descending section of the ionizing shock adiabatic appears due to the fact that there is a range of sufficiently large Mach numbers M_1 for which the plasma behind the relaxation zone becomes practically fully ionized with respect to the first ionization, while the degree of second ionization still remains small (in the calculating model the latter is neglected). As a result, a subsequent increase of M_1 leads to more rapid growth of the equilibrium temperature T_2 than in the range of lower values of M_1, so that the equilibrium gas density ρ_2 starts to decrease. Analogous mechanisms for the formation of nonmonotonic shock adiabatics is displayed in dissociating shocks propagating in molecular gases. Examples of such adiabatics may be found in Refs. 2 and 9.

The experimental data for argon presented in Fig. 2 correspond to the increasing section of the calculated dependence $\eta(M_1)$. Griffiths et al.[9] went beyond that limit, and indeed their experimental results indicate a maximum compression ratio $\eta_m \approx 14$ for argon at 1 Torr, occurring at a pressure ratio $P_m \approx 1500$. These results correspond to those obtained by Eqs. (1–3).

Acknowledgment

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References

[1]Liberman, M. A., and Velikovich, A. L., *Physics of Shock Waves in Gases and Plasmas*, Springer-Verlag, Berlin, 1986.

[2]Anderson, J. D., *Hypersonic and High Temperature Gas Dynamics*, McGraw-Hill, New York, 1989.

[3]Tsikulin, M. A., and Popov, E. G., *Radiative Properties of Shock Waves in Gases*, Moscow, Nauka, 1977 (in Russian).

[4]Asinovsky, E. I., Lebedev, E. F., and Ostashev, V. E., "Investigation of Processes Determining the Efficiency of Energy Conversion in a Linear Explosive MHD Generator," *Proceedings of the 7th International Conference on MHD Electrical Power Generation* (Cambridge, MA), Vol. 2, 1980, pp. 605–612.

[5]Lukyanov, G. A., *Supersonic Plasma Jets*, Mashinostroenie, Leningrad, 1985 (in Russian).

[6]Pinegre, M., These, Universite de Rouen, Mont-Saint-Aignan, France, 1976.

[7]Glass, I. I., and Liu, W. S., "Effects of Hydrogen Impurities on Shock Structure and Stability in Ionizing Monatomic Gases. Part 1. Argon," *Journal of Fluid Mechanics*, Vol. 84, 1978, pp. 55–77.

[8]Kaniel, A., Igra, O., Ben-Dor, G., and Mond, M., "Ionization Behind Strong Shocks in Argon," *Physics of Fluids*, Vol. 29, No. 11, 1986, pp. 3618–3625.

[9]Griffiths, R. W., Sandeman, R. J., and Hornung, H. G., "The Stability of Shock Waves in Ionizing and Dissociating Gases," *Journal of Physics D: Applied Physics*, Vol. 9, 1976, pp. 1681–1691.

Solid Rocket Motor Temperature Sensitivity

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Introduction

TEMPERATURE sensitivity refers to the influence of the initial propellant temperature on the propellant burning rate and encompasses the effect of initial temperature on the thrust and burning time of the solid propellant rocket motor. The temperature sensitivity of the propellant and the solid rocket motor are described by several different temperature sensitivity coefficients.[1] The coefficients are, the temperature sensitivity of the burning rate at constant combustion chamber pressure (a propellant composition parameter only)

$$\sigma_p = \left(\frac{\partial \ln r}{\partial T}\right)_p \tag{1}$$

the temperature sensitivity of burning rate at constant motor geometry, a constant value of the propellant area ratio[2] K (both a rocket motor parameter and a propellant composition parameter)

$$\sigma_K = \left(\frac{\partial \ln r}{\partial T}\right)_K \tag{2}$$

and the temperature sensitivity of the combustion chamber pressure for constant motor geometry (both a propellant composition parameter and a rocket motor parameter):

$$\pi_K = \left(\frac{\partial \ln p}{\partial T}\right)_K \tag{3}$$

Another coefficient, the temperature sensitivity of the characteristic velocity for constant motor geometry (both a propellant composition parameter and a rocket motor parameter)

$$\pi_C = \left(\frac{\partial \ln c^*}{\partial T}\right)_K \tag{4}$$

is necessary for deriving a working relationship for Eq. (3) since the characteristic velocity is involved in the equilibrium combustion chamber pressure equation.[2]

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